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THE ENVELOPE OF RANDOM VIBRATION OF A

LIGHTLY-DAMPED NONLINEAR OSCILLATOR

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When a lightly-damped oscillator is excited by a stationary random
process the response has the appearance of an oscillation with a slow,
random fluctuation in amplitude. The envelope of the oscillation has
been studied by Rice [1] for the case of stationary Gaussian response.
When Gaussian excitation is applied to a <u>nonlinear</u> oscillator the
response is generally non-Gaussian. In the non-Gaussian case the Rice
technique for defining an envelope is no longer directly applicable.
It is not clear whether there is any possibility of extending the Rice
approach to obtain meaningful results in the non-Gaussian case. The
present author has given two alternative techniques [2] which permit
evaluation of the first-order probability distribution for the response
envelope of a nonlinear single-degree-of-freedom oscillator with
Gaussian excitation. The purpose of this note is to show that for a
general class of nonlinear restoring forces these two first-order
distributions are identical. Furthermore, for <u>linear</u> systems this
distribution is identical with that given by the Rice approach. Our

ABSTRACT

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There are several ways in which the envelope of the stationary random response of a lightly-damped oscillator can be characterized. Two characterizations which lead to the same first-order probability distributions for systems with conservative nonlinear restoring forces are described. In such systems the envelope distributions are in general different from the distributions of peak amplitudes. The Duffing system is used to illustrate these general results.

For the special case of a Gaussian oscillator an envelope characterization has been given by Rice. It is shown that in this case the above envelope characterization has the same first-order distribution as that given by Rice, but that the second-order probability distributions are, in general, different.

envelope definition is never-the-less not completely identical with that of Rice even for narrow-band Gaussian responses because the second-order probability distributions for an ideal band-pass filter differ by terms proportional to the square of the bandwidth.

1. Envelope Obtained from Peak Statistics

We consider nonlinear oscillators described by the equation

$$\ddot{x} + \beta \dot{x} + g(x) = f(v) \tag{1}$$

where β is a constant linear damping coefficient, $g(x)$ is the nonlinear restoring force function and $f(t)$ is the excitation which we will take to be a stationary Gaussian process with uniform spectral density S_0 (acceleration squared per unit of circular frequency) over all frequencies from $\omega = -\infty$ to $\omega = \infty$. We will assume that $g(x)$ is an odd function such that $xg(x)$ is positive definite. The integral

$$G(x) = \int_0^x g(\xi) \, d\xi \tag{2}$$

is thus a positive definite even function. It is further appropriate to assume that $g(x)$ remains sufficiently large for large $\underline{x}$ that

$$\int_{-\infty}^{\infty} x^2 \, e^{-\frac{G(x)}{\sigma_v^2}} \, dx < \infty \tag{3}$$

where σ_v is a real constant. The condition (3) insures that the stationary response $x(t)$ of (1) has a finite mean square.

The joint probability distribution density $p(x,v)$ of $\underline{x}$ and $v = \dot{x}$ for the stationary response of (1) can be obtained as a solution of the

Fokker-Planck equation $\begin{bmatrix}3\end{bmatrix}$. We find

$$p(x,v) = C \, exp\left\{ -\frac{1}{\tilde{\sigma}_v^2}\left[\frac{v^2}{2} + G(x)\right]\right\}$$ (4)

where C is the normalization constant and

$$\tilde{\sigma}_v^2 = \frac{\pi S_o}{\beta}$$ (5)

is the mean square value of v(t) in the stationary response. The expected number of crossings of the level x = a with positive slope per unit time ν_a^+ is $\begin{bmatrix}1\end{bmatrix}$

$$\nu_a^+ = \int_o^\infty v \, p(a,v) \, dv$$ (6)

Inserting (4) into (6) yields

$$\nu_a^+ = \nu_o^+ \, e^{-\frac{G(a)}{\tilde{\sigma}_v^2}}$$ (7)

where ν_o^+ is the expected number of zero-crossings with positive slope per unit time. The statistical parameter (7) can also be considered to represent a probability distribution by the following heuristic argument. In a narrow-band process the response consists essentially of recognizable cycles with one positive peak and one negative peak per cycle. The parameter ν_o^+ counts the average number of cycles per unit time while the parameter ν_a^+ counts the average number of cycles per unit time with peaks above the level x = a. Thus, on the average, the fraction of cycles

with peaks higher than x = a is

$$P(peak > a) = \frac{\nu_a^+}{\nu_o^+} \tag{8}$$

The probability __density__ of peaks is

$$p_p(a) = -\frac{dP}{da} = -\frac{1}{\nu_o^+}\frac{d\nu_a^+}{da} = \frac{g(a)}{\sigma_v^2} e^{-\frac{G(a)}{\sigma_v^2}} \tag{9}$$

substituting from (7) and (2). It is to be emphasized that (8) and (9) represent the distribution of peaks within the population of all positive peaks.

An envelope distribution can be obtained from (9) by assuming that the envelope is a smooth gradual curve joining the peaks and that on the average the time the envelope spends between a and a + da is just the number of such peaks multiplied by τ(a) which is the undamped period of a free vibration of amplitude a. The fraction of time so spent is

$$p_e(a)\,da = p_p(a)\,\nu_o^+\,\tau(a)\,da \tag{10}$$

The undamped period τ(a) can be obtained by integrating (1) with $\beta = 0$ and f(t) = 0.

$$\tau(a) = 4\int_o^a \frac{dx}{\sqrt{2[G(a)-G(x)]}} \tag{11}$$

Inserting (9) and (11) into (10) we have the first-order probability density for an envelope of the response process defined by (1).

$$p_e(a) = \frac{4 \nu_o^+ g(a)}{\sigma_v^2} e^{-\frac{G(a)}{\sigma_v^2}} \int_0^a \frac{dx}{\sqrt{2\left[G(a) - G(x)\right]}}$$

(12)

Evaluation of (12) for a particular nonlinear function g(x) may be difficult. In addition to the operations explicitly indicated in (12) the parameter ν_o^+ requires evaluation of the normalization constant in (4) as follows

$$\nu_o^+ = C \sigma_v^2 = \frac{\sigma_v}{\sqrt{2\pi}} \left(\int_{-\infty}^{\infty} e^{-\frac{G(x)}{\sigma_v^2}} dx \right)^{-1}$$

(13)

The arguments leading to (12) are not rigorous. Furthermore, the envelope so defined is insufficiently detailed to permit extension beyond the first-order probability distribution and its accompanying first-order statistics. We next consider a more definitely defined envelope which turns out to have the same first-order distribution as (12).

3. Envelope defined by total energy

Along with the random processes x(t) and v(t) determined by (1) let us consider the envelope process a(t) where

$$G(a) = \frac{v^2}{2} + G(x)$$

(14)

The interpretation here is that a(t) is the amplitude which would be attained if the sum of the instantaneous kinetic and potential energies were converted entirely into potential energy; i.e., a(t) is the peak amplitude which would be obtained in a hypothetical free undamped oscillation of the system if the initial conditions for the free oscillation were the current values of x(t) and v(t). The first-order probability distribution for a(t) then follows from the joint distribution of $\underline{x}$ and $\underline{v}$. Taking advantage of symmetry we have

$$P(\text{envelope} < a) = 4 \int_0^a dx \int_0^{\sqrt{2[G(a) - G(x)]}} p(x, v) \, dv \qquad (15)$$

as the ensemble fraction of cases for which the combination of $\underline{x}$ and $\underline{v}$ makes the envelope defined by (14) smaller than a given value of $\underline{a}$. The probability density for the envelope is

$$p_e(a) = \frac{dP}{da} = 4 \int_0^a p(x, \sqrt{2[G(a) - G(x)]}) \frac{g(a) \, dx}{\sqrt{2[G(a) - G(x)]}} \qquad (16)$$

Inserting the joint density (4) leads to

$$p_e(a) = 4C \, g(a) \, e^{-\frac{G(a)}{\sigma_v^2}} \int_0^a \frac{dx}{\sqrt{2[G(a) - G(x)]}} \qquad (17)$$

which is identical with (12) if we take into account the relation (13) between the normalization constant C and the average frequency ν_0^+. Thus for arbitrary nonlinear restoring forces of the type indicated the heuristic envelope of Section 1 has the same first-order probability distribution as the envelope based on the energy relation (14).

3. Example - Duffing System

We consider the particular case of (1) in which

$$g(x) = \omega_o^2 x \left(1 + \frac{x^2}{L^2} \right) \tag{18}$$

where it is possible to evaluate the general expressions obtained above. A convenient normalization is provided by the parameter σ_o where

$$\sigma_o^2 = \frac{\pi S_o}{\beta \omega_o^2} \tag{19}$$

is the mean square value of $\underline{x}$ in the limiting linear case where $L \to \infty$. Setting

$$y = \frac{x}{\sigma_o} \qquad\qquad \epsilon = \frac{\sigma_o}{L} \tag{20}$$

the governing equation (1) takes the form

$$\ddot{y} + \beta \dot{y} + \omega_o^2 y \left(1 + \epsilon^2 y^2 \right) = \frac{f(t)}{\sigma_o} \tag{21}$$

The free undamped oscillations of amplitude $y_{max} = a$ can be described in terms of the Jacobian elliptic cosine function [4]

$$y = a \, cn \left(\omega_1 t, \, m \right) \tag{22}$$

where $\underline{m}$ is the modulus of the elliptic function

$$m = \frac{\epsilon^2 a^4}{2 \left(1 + \epsilon^2 a^2 \right)} \tag{23}$$

and the parameter ω_1 is given by

$$\omega_1^2 = \omega_o^2 \left(1 + \epsilon^2 a^2 \right) \tag{24}$$

The period $\tau(a)$ is

$$\tau(a) = \frac{4}{\omega_1} K(m) \qquad (25)$$

where K(m) is the complete elliptic integral which gives the real quarter period of the elliptic function [4].

The joint probability $p(y,\dot{y})$ for this case follows from (4)

$$p(y,\dot{y}) = \frac{1}{\omega_0 I(\epsilon)} \exp\left\{ -\frac{\dot{y}^2}{2\omega_0^2} - \frac{y^2}{2}\left(1 + \frac{1}{2}\epsilon^2 y^2\right)\right\} \qquad (26)$$

where the integral

$$I(\epsilon) = 2\sqrt{2\pi} \int_0^\infty e^{-\frac{y^2}{2}\left(1 + \frac{1}{2}\epsilon^2 y^2\right)} dy \qquad (27)$$

provides the correct normalization. The probability density for the peaks (9) is obtained without evaluating (27)

$$p_p(a) = a(1 + \epsilon^2 a^2) e^{-\frac{a^2}{2}\left(1 + \frac{1}{2}\epsilon^2 a^2\right)} \qquad (28)$$

and is plotted in Fig. 1 for several values of the nonlinearity parameter ϵ. This distribution was recently obtained by Lyon [5] using essentially the same procedure as that described in Section 1. In [5] it was additionally suggested that (28) might also be interpreted as the density function of an envelope. Since this suggestion takes no account of the tendency of "cycles" with different peak amplitudes to have different periods we doubt that it

portrays an envelope as effectively as the distribution

$$p_e(a) = \nu_0^+ \, \tau(a) \, p_p(a)$$

$$= \frac{4 \, K(m)}{T(\epsilon) \sqrt{1 + \epsilon^2 a^2}} \, p_p(a) \tag{29}$$

which follows from (10) by using (25) and the relation

$$\nu_0^+ = \frac{\omega_0}{T(\epsilon)} \tag{30}$$

which is obtained from (13). The distribution (29) can also be obtained directly from (17) when the integral is recognized as the complete elliptic integral of (25). Fig. 2 shows plots of (29) for different values of ϵ.

For the Duffing oscillator there is not a great difference between (28) and (29) until ϵ is quite large. In Fig. 3 there is a comparison between the distributions of (28) and (29) for $\epsilon = 2$. When $\epsilon = 0$ the system (21) is linear, the undamped free vibration period is independent of amplitude and the distributions (28) and (29) both reduce to the Rayeligh distribution

$$p(a) = a \, e^{-\frac{a^2}{2}} \tag{31}$$

which is the known amplitude distribution for narrow-band Gaussian processes and serves equally well to describe the peaks or the envelope.

The first-order distribution of the response y is of some auxiliary interest for this system. It follows from (26) by integration over all

values of $\dot{y}$.

$$p(y) = \frac{\sqrt{2\pi}}{I(\epsilon)} \; e^{-\frac{y^2}{2}(1 + \frac{1}{2}\epsilon^2 y^2)} \tag{32}$$

This is shown in Figure 4 for several values of the nonlinearity parameter
ϵ .

4. Second-order Envelope Distribution

The energy-based envelope definition (14) gives promise of providing
a means for obtaining second-order probabilities or second-order statistics
for the envelope of the response of (1) to Gaussian excitation. The joint
probability that at time $\underline{t}$ the envelope will be less than a_1 and that at
time $t + \mathcal{T}$ the envelope will be less than a_2 is given by

$$P = \int_{-a_1}^{a_1} dx_1 \int_{-\sqrt{2[G(a_1)-G(x_1)]}}^{\sqrt{2[G(a_1)-G(x_1)]}} d\,v_1 \int_{-a_2}^{a_2} dx_2 \int_{\sqrt{2[G(a_2)-G(x_2)]}}^{\sqrt{2[G(a_2)-G(x_2)]}} p(x_1, v_1, x_2, v_2)dv_2 \tag{33}$$

as a direct extension of (15). The density $p(a_1,a_2)$ then follows from
differentiation of (33). Our work along this line has so far been
limited to cases where $p(x_1,v_1,x_2,v_2)$ has been Gaussian. In the Gaussian
case we will show that the second-order density $p(a_1,a_2)$ so obtained is
not identical with the corresponding density which follows from Rice's
envelope definition.

When the system (1) is linear the response to Gaussian excitation
is also Gaussian. In this case the envelope definition (14) can be altered
to make it depend solely on the response process; i.e., independent of the

oscillator parameters. Thus for a narrow-band Gaussian processes $x(t)$ the energy-based envelope $a(t)$ is given by

$$a^2 = x^2 + \left(\frac{v}{\omega_0}\right)^2 \qquad (34)$$

where $\omega_0 = 2\pi\nu_0^+$ is the expected circular frequency of the process and $v = \dot{x}$. The result (34) follows from setting $G(x) = (1/2)\omega_0^2 x^2$ in (14). The first-order distribution of $a(t)$ is that of Rayleigh as noted in the previous section. The second order density follows from (33) but because of (34) a great simplification is obtained by transforming to polar coordinates. We find

$$p(a_1, a_2) = \int_0^{2\pi} a_1 \, d\varphi_1 \int_0^{2\pi} a_2 \, d\varphi_2 \; p(a_1 \sin\varphi_1, a_1\omega_0\cos\varphi_1, a_2\sin\varphi_2, a_2\omega_0\cos\varphi_2) \quad (35)$$

In order to evaluate this it is necessary to know the joint density $p(x_1, v_1, x_2, v_2)$ which in turn depends on the autocorrelation function

$$R(\tau) = E[x_1 x_2] \qquad (36)$$

and its first two derivatives. We shall limit our further discussion to a particular choice for (36).

5. Example-Ideal Band Pass Filter

Let the spectral density of $x(t)$ be zero except in bands of width 2α centered on $\pm\omega_m$ where the density is constant. The corresponding autocorrelation function is

$$R(\tau) = \sigma^2 \frac{\sin\alpha\tau}{\alpha\tau} \cos\omega_m\tau \qquad (37)$$

where $\sigma^2 = E[x^2]$ is the mean square of the process. We introduce the following notation

$$f_1(\varphi) = \frac{\sin \varphi}{\varphi}$$

$$f_2(\varphi) = 3 \frac{f_1(\varphi) - \cos \varphi}{\varphi^2}$$

$$\varsigma = \frac{1}{\sqrt{3}} \frac{\alpha}{\omega_m}$$

(38)

The functions f_1 and f_2 are shown in Figure 5. The parameter ς is a measure of the process bandwidth. In terms of these we find successively

$$R(\tau) = \sigma^2 f_1(\alpha\tau) \cos \omega_m \tau$$

$$-R'(\tau) = \sigma^2 \omega_m \left[f_1(\alpha\tau) \sin \omega_m \tau + \varsigma^2 f_2(\alpha\tau) \omega_m \tau \cos \omega_m \tau \right]$$

$$-R''(\tau) = \sigma^2 \omega_m^2 \left\{ \left[(1+3\varsigma^2) f_1(\alpha\tau) - 2\varsigma^2 f_2(\alpha\tau)\right] \cos \omega_m \tau - 2\varsigma^2 f_2(\alpha\tau) \omega_m \tau \sin \omega_m \tau \right\}$$

(39)

The joint density of four Gaussian variables y_1, y_2, y_3 and y_4 _having zero means_ is

$$p(y_1, y_2, y_3, y_4) = \frac{1}{4\pi^2 (\Lambda)^{1/2}} \exp\left[-\frac{1}{2\Lambda} \left(\sum_i^4 \sum_j^4 \Lambda_{ij} y_i y_j \right) \right]$$

(40)

where Λ is the determinant and the Λ_{ij} are the cofactors of the covariance matrix for the y_i. If we identify the y_i with x_1, v_1, x_2 and v_2 respectively the covariance matrix is

$$\begin{bmatrix} R(0) & 0 & R(\tau) & R'(\tau) \\ 0 & -R''(0) & -R'(\tau) & -R''(\tau) \\ R(\tau) & -R'(\tau) & R(0) & 0 \\ R'(\tau) & -R''(\tau) & 0 & -R''(0) \end{bmatrix}$$

(41)

In principle all that remains is to insert (39) into (41), evaluate (40) and then obtain the second-order probability distribution density using (35). We have, however, been unable to complete this program without approximation; but if we neglect terms which are $O(\zeta^2)$ in (39) the calculation is rather simple. For the distribution (40) we find

$$p(x_1, v_1, x_2, v_2) = \frac{1}{4\pi^2\sigma^4\omega_m^2(1-f_1^2)} \exp\left[\frac{-N}{2\sigma^2\omega_m^2(1-f_1^2)}\right] \qquad (42)$$

with

$$N = \omega_m^2(x_1^2+x_2^2) + v_1^2+v_2^2 - 2f_1\cos\omega_m\tau\,(v_1 v_2 + \omega_m^2 x_1 x_2)$$
$$+ 2\omega_m f_1 \sin\omega_m\tau\,(x_1 v_2 - x_2 v_1) \qquad (43)$$

Here the argument of f_1 is $\propto\tau$ as in (39).

A final preliminary before evaluating (35) is the determination of the expected frequency ω_0.

$$\omega_0^2 = \frac{-R''(\omega)}{R(0)} = \omega_m^2(1+\zeta^2) \qquad (44)$$

Here again we have neglected the ζ^2 term and taken ω_0^2 to be identical with ω_m^2. The integration of (32) then becomes straightforward and we obtain for the joint density

$$p(a_1, a_2) = \frac{a_1 a_2}{\sigma^4(1-f_1^2)} \, e^{-\frac{a_1^2+a_2^2}{2\sigma^2(1-f_1^2)}} I_0\left[\frac{a_1 a_2 f_1}{\sigma^2(1-f_1^2)}\right] \qquad (45)$$

which is identical with that obtained by Rice [1] with a different envelope definition. Since we have had to make $O(\zeta^2)$ approximations whereas no such approximation was required in [1] there is a suggestion that the two envelopes are not the same.

In order to establish this point we turn to a simple second-order envelope statistic which can be evaluated exactly for both envelope definitions. Using Rice's definition and hence (45)

$$E[a_1^2 a_2^2] = \int_0^\infty a_1^2 \, da_1 \int_0^\infty a_2^2 \, p(a_1, a_2) \, da_2$$

(46)

$$= 4 \sigma^4 [1 + f_1^2(\alpha \tau)]$$

where the integration is performed along the lines indicated in [6] and use is made of [7], [8] and [9].

Using the definition (34) we have

$$E[a_1^2 a_2^2] = E[x_1^2 x_2^2] + \frac{1}{\omega_0^2}\{E[x_1^2 v_2^2] + E[x_2^2 v_1^2]\} + \frac{1}{\omega_0^4} E[v_1^2 v_2^2]$$

(47)

and since x_1, x_2, v_1 and v_2 are jointly Gaussian

$$E[x_1^2 x_2^2] = \sigma^4 + 2[R(\tau)]^2$$

$$E[x_1^2 v_2^2] = E[x_2^2 v_1^2] = \omega_0^2 \sigma^4 + 2[R'(\tau)]^2$$

(48)

$$E[v_1^2 v_2^2] = \omega_0^4 \sigma^4 + 2[R''(\tau)]^2$$

so that using (39) permits us to evaluate (47) exactly. The result is

fairly complicated but it can be put in the following form

$$E\left[a_1^2 a_2^2\right] = 4\sigma^4\left[1 + f_1^2(\alpha\tau)\right] + \zeta^2 f_3(\tau) + O(\zeta^4) \quad (49)$$

with

$$f_3(\tau) = 4\sigma^4\left[(3\cos^2\omega_m\tau - 1)f_1^2(\alpha\tau) - 2\cos\omega_m\tau\left(\cos\omega_m\tau - \omega_m\tau\sin\omega_m\tau\right)f_1(\alpha\tau)f_2(\alpha\tau)\right] \quad (50)$$

which indicates that there definitely is an $O(\zeta^2)$ discrepancy between the values of $E\left[a_1^2 \, a_2^2\right]$ obtained from Rice's envelope definition and from the energy-based envelope (34). Finally it should be noted that this discrepancy applies only to the special ideal band-pass process of (37). For other narrow-band processes the discrepancy may not necessarily be proportional to the square of the bandwidth.

ACKNOWLEDGMENT

REFERENCES

1. S. O. Rice, Mathematical analysis of random noise, Bell System Tech. J., 23, 282-332 (1944); 24, 46-156 (1945). Reprinted in N. Wax, "Selected Papers on Noise and Stochastic Processes," Dover Publications, N.Y., (1954).

2. S.H. Crandall, Random vibration of a nonlinear system with a set-up spring, ASME Paper No. 61-WA-152, to appear in J. Appl. Mechs.

3. See, for example, S.H. Crandall, Random vibration of systems with nonlinear restoring forces, to appear in the Proceedings of the International Symposium on Nonlinear Oscillations, Kiev, September, 1961, also available as AFOSR 708, June, 1961.

4. L.M. Milne-Thomson, "Jacobian Elliptic Function Tables," Dover Publications, Inc., N.Y., 1950.

5. R.H. Lyon, On the Vibration Statistics of a Randomly Excited Hand-Spring Oscillator, J. Acoust. Soc. Am., 32, 716-719 (1960).

6. R.H. Lyon, On the Vibration Statistics of a Randomly Excited Hand-Spring Oscillator, II, J. Acoust. Soc. Am., 33, 1395-1403, (1961).

7. A. Erdelyi, Bateman Manuscript Project, "Higher Transcendental Functions, Vol. II," McGraw-Hill Book Co., N.Y., 1953. Formula (15) on p. 18.

8. Reference 7. Formula (35) on p. 93.

9. A. Erdelyi, Bateman Manuscript Project, "Higher Transcendental Functions, Vol. I," McGraw-Hill Book Co., N.Y., 1953. Formula (7) on p. 101.

CAPTIONS FOR FIGURES

1. Probability density distribution of peaks for Duffing oscillator subjected to white-noise excitation for various values of the nonlinearity parameter.

2. Probability density distribution of an envelope for the Duffing oscillator.

3. Comparison of probability density distributions of the peaks (28) and of the envelope (29) for the Duffing oscillator with $\epsilon = 2$.

4. Probability density distribution for Duffing system response process.

5. The functions of $f_1(\varphi)$ and $f_2(\varphi)$ defined in (38) and used to describe the correlations for the ideal band-pass filter response.
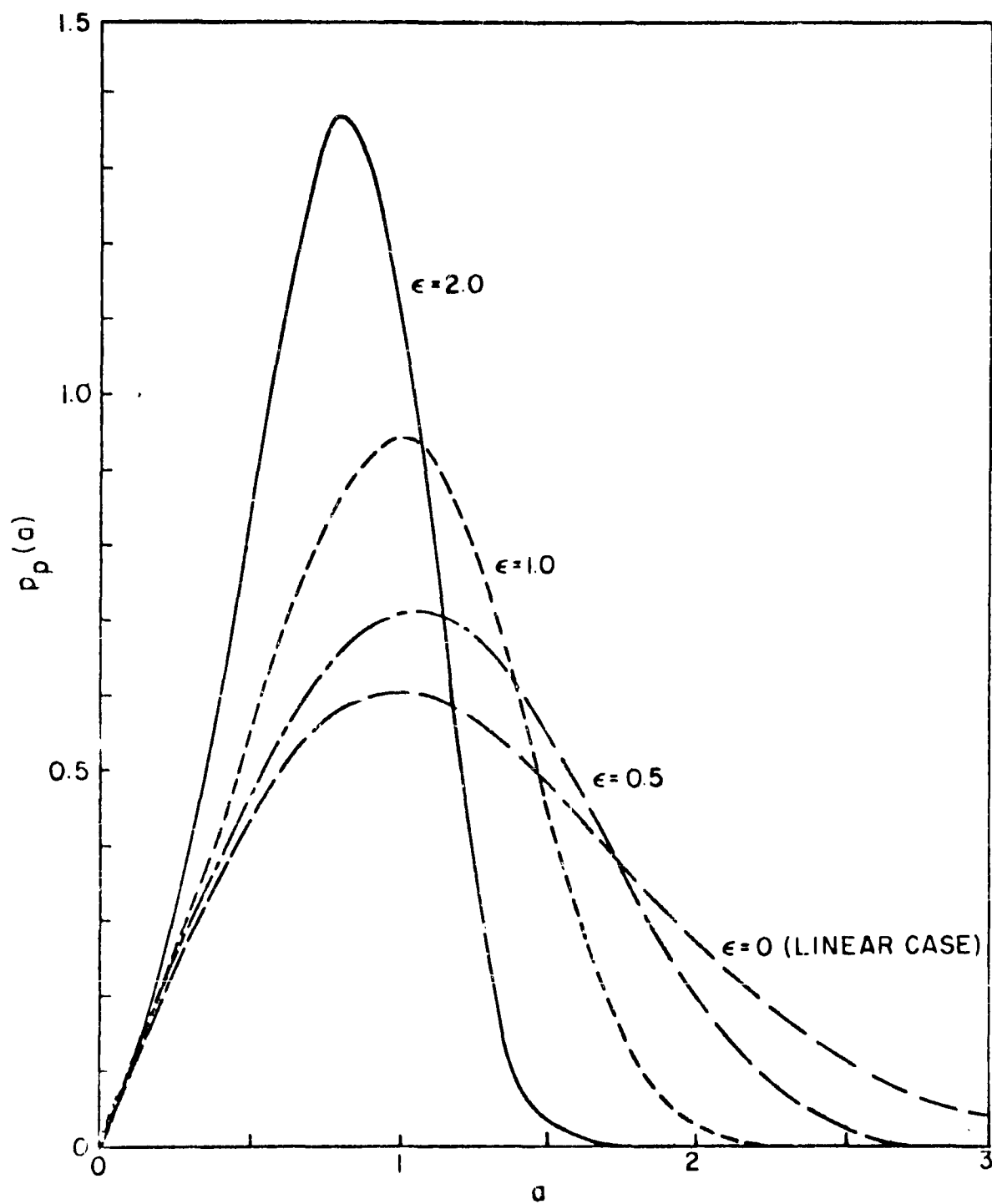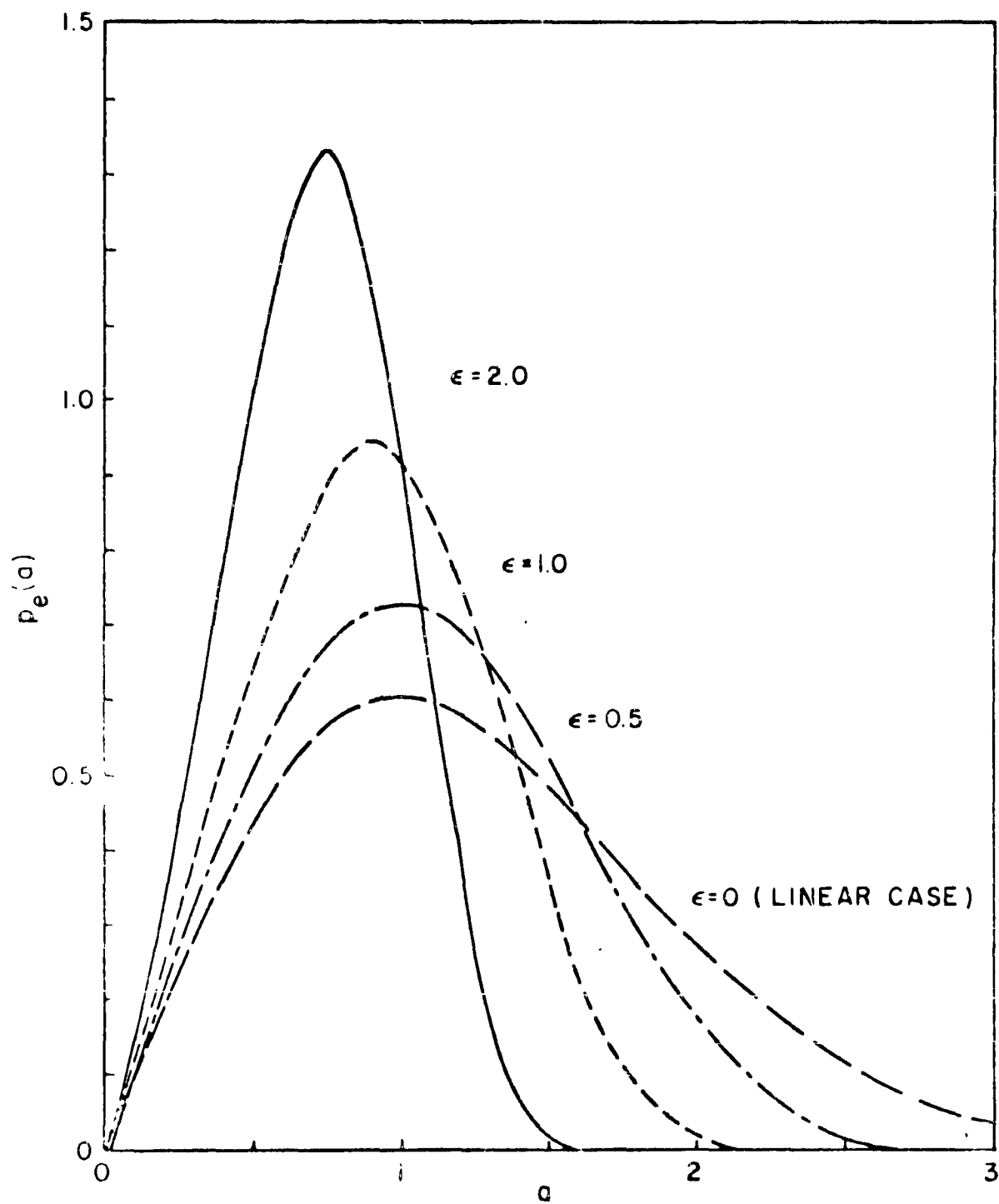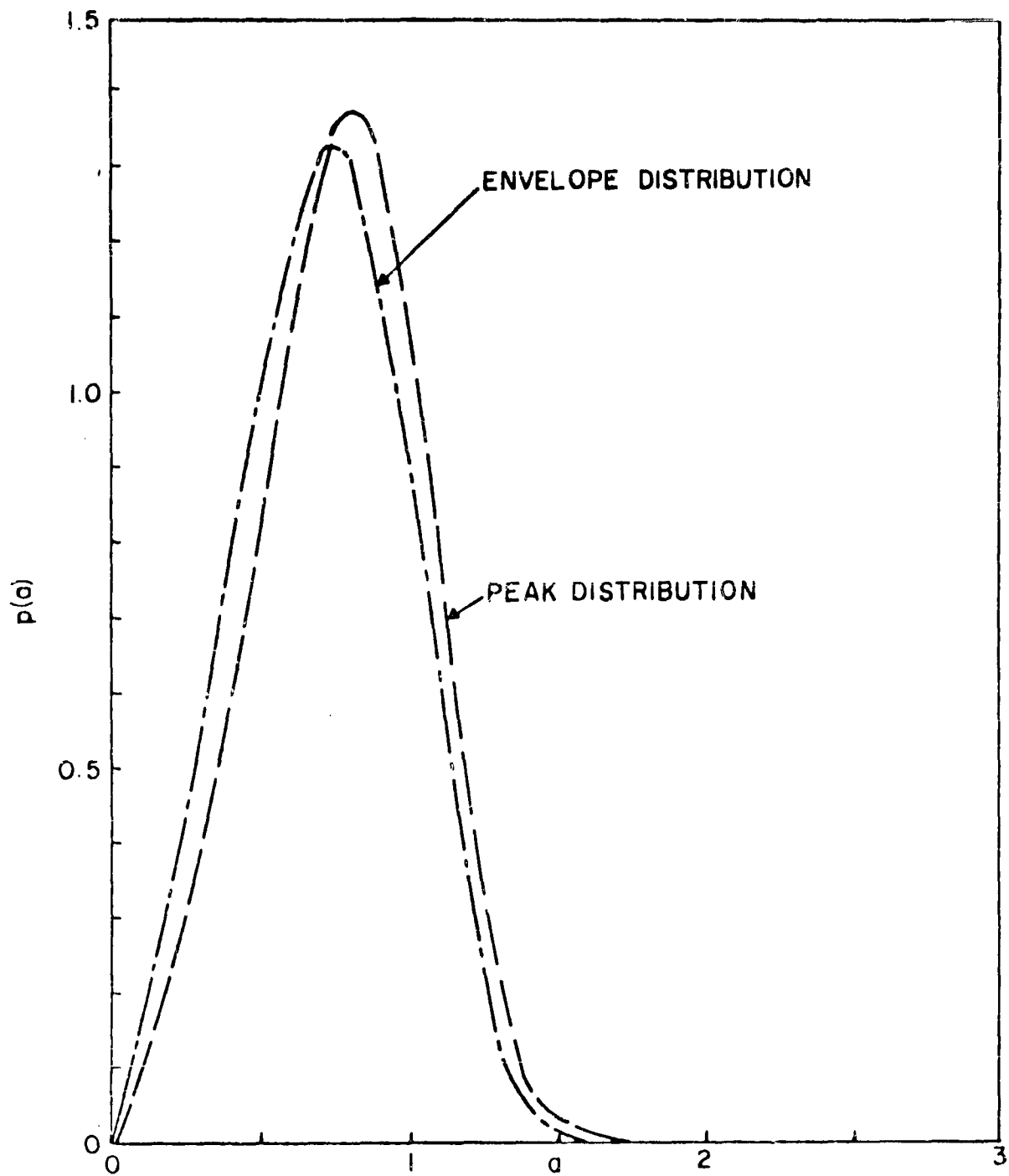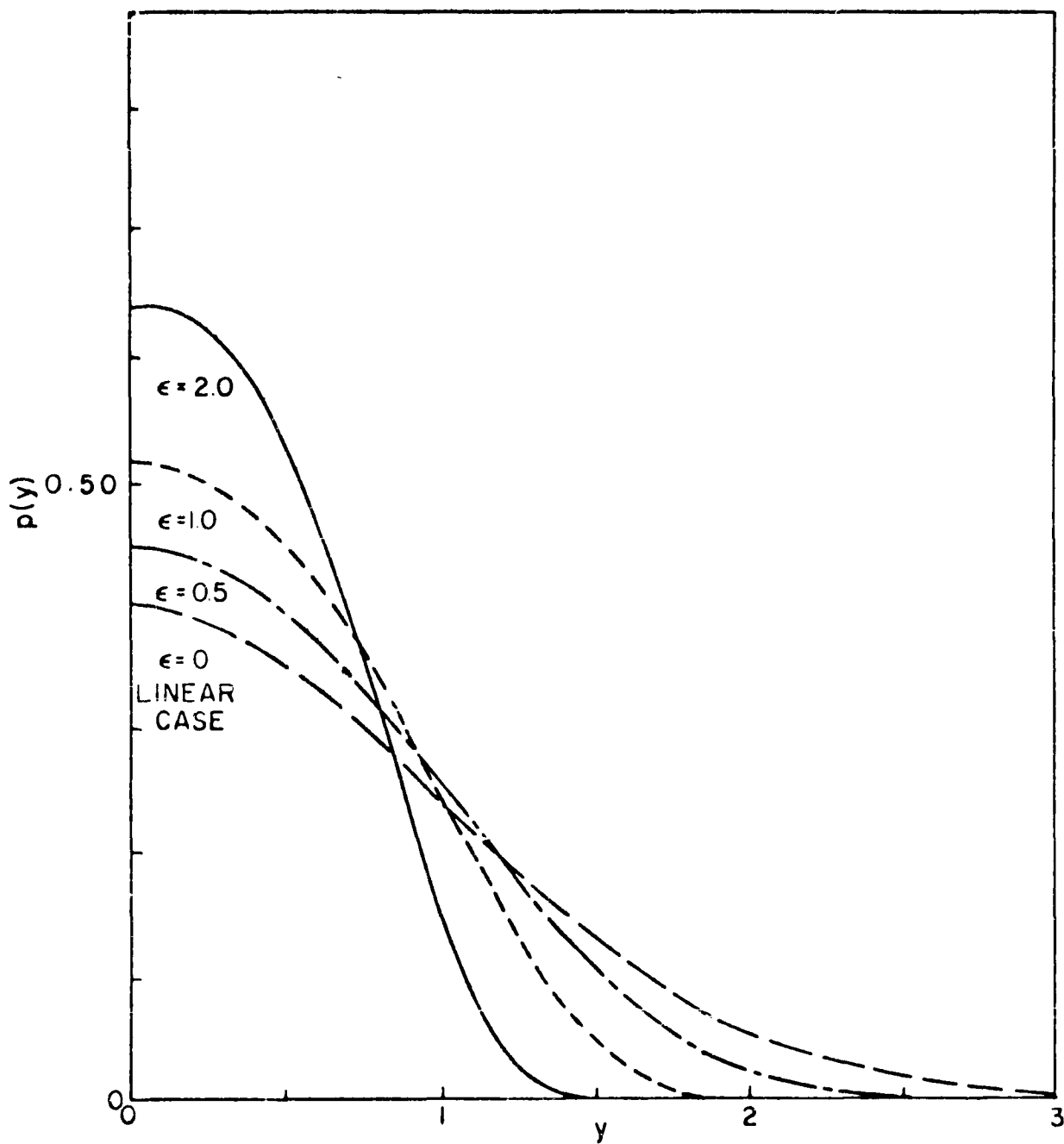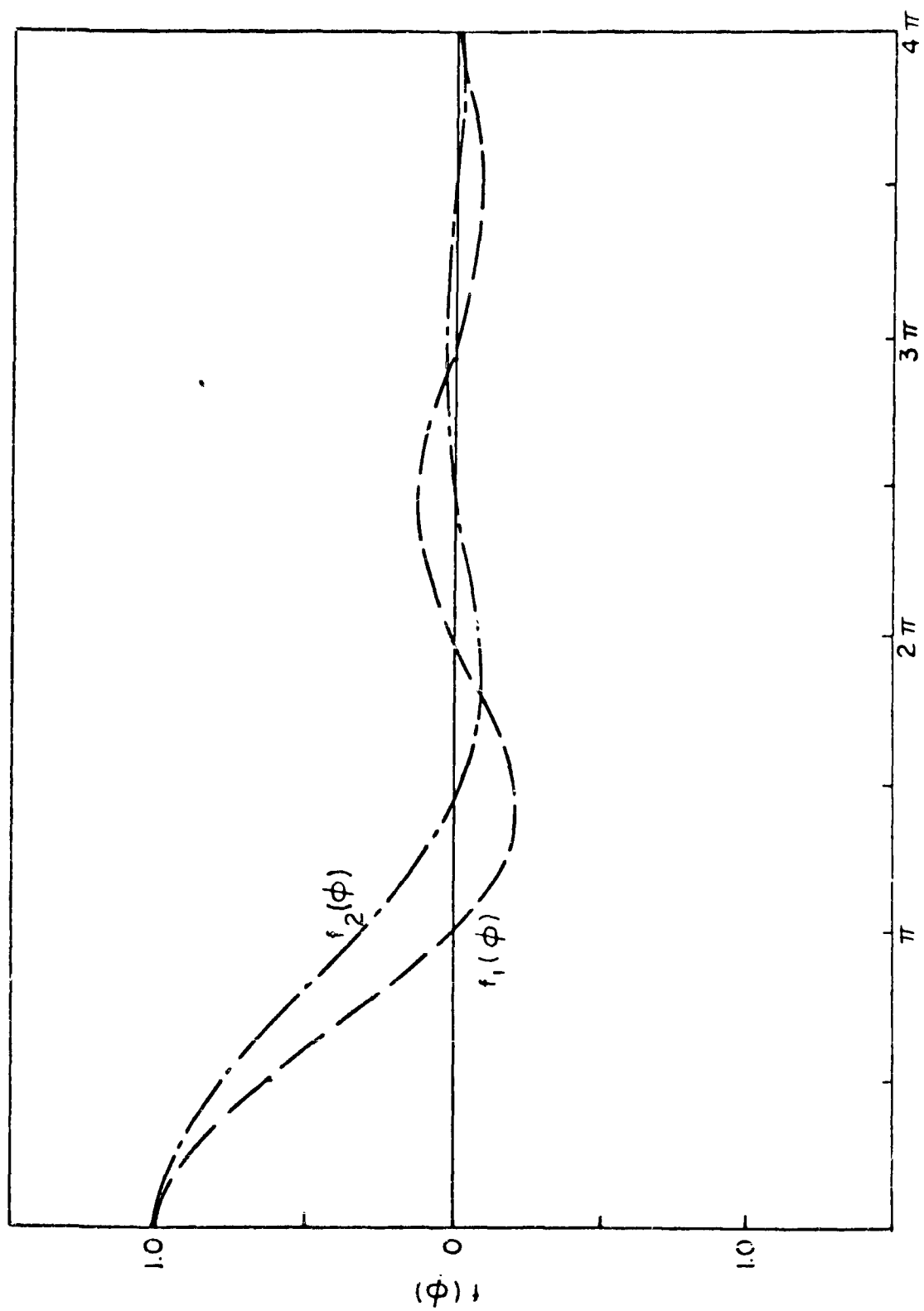
FIGURE I

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

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